'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 4th Semester Examination, 2023

GE2-P2-MATHEMATICS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains MATHGE4-II, MATHGE4-III, MATHGE4-V. The candidates are required to answer any *one* from the *three* courses. Candidates should mention it clearly on the Answer Book.

MATHGE4-II

ALGEBRA

GROUP-A

1. Answer any ***four*** questions: $3\times4 = 12$

(a) Prove that $1+2+\cdots\cdots+n=\dfrac{n(n+1)}{2}$, $\forall\, n\in\mathbb{N}$, by the principle of induction. 3

(b) Show that the product of all values of $(1+i\sqrt{3})^{3/4}$ is 8. 3

(c) Prove that an inverse of the equivalence relation is an equivalence relation. 3

(d) Find the number and nature of real roots of the equation: 3

$$x^4+4x^3-x^2-2x-5=0\,.$$

(e) Find all eigen values of the following matrix: 3

$$\begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}.$$

(f) Find the rank of the matrix $\begin{pmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 1 & 4 & 4 \end{pmatrix}$. 3

GROUP-B

Answer any *four* questions $6\times4 = 24$

2. Prove that $a^6+b^6+c^6+d^6 \ge abcd\,(a^2+b^2+c^2+d^2)$ where a, b, c, d are four positive real numbers. 6

3. Using the principle of induction, prove that $10^{n+1}+10^n+1$ is divisible by 3 for all $n\in\mathbb{N}$. 6

4. Solve the system of equations using row reduced form: 6

$$x+2y+z=1$$
$$3x+y+2z=3$$
$$x+7y+2z=1$$

5. Verify Cayley-Hamilton theorem for the following matrix: 6

$$A=\begin{pmatrix}1 & 0 & 2\\ 0 & -1 & 1\\ 0 & 1 & 0\end{pmatrix}$$

Hence find A^{-1} and A^9.

6. Solve: $2x^4+8x^3+3x^2+4x+1=0$, whose the sum of two roots is zero. 6

7. Solve by Ferrari's method: 6

$$x^4+4x^3-6x^2+20x+8=0$$

GROUP-C

Answer any *two* questions $12\times 2=24$

8. (a) If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, find the equation whose roots are $\beta^2+\gamma^2-\alpha^2,\ \gamma^2+\alpha^2-\beta^2,\ \alpha^2+\beta^2-\gamma^2$. 3

(b) Find the least positive residue in $2^{41}(\text{mod } 23)$. 3

(c) Show that $a^{12}-b^{12}$ is divisible by 91 if *a* and *b* both are prime to 91. 6

9. (a) Reduce the matrix *A* to its row reduced echelon form and hence find its rank, 4

where $A=\begin{pmatrix}0 & 1 & -3 & 1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{pmatrix}$.

(b) If *p* is a prime and *a*, *b* are positive integers, then show that 4

$$(a+b)^p \equiv (a+b)\ (\text{mod } p).$$

(c) Find two integers *u* and *v* satisfying $54u+24v=30$. 4

10.(a) Solve $x^3-6x^2-6x-7=0$ by Cardon's method. 6

(b) If $\cos\alpha+\cos\beta+\cos\gamma=0$ and $\sin\alpha+\sin\beta+\sin\gamma=0$, prove that 6

$$\cos 3\alpha+\cos 3\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$$

and $$\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$$

11.(a) Find $\cos 2\theta \cosh 2\phi$, if $\sin(\theta+i\phi)=\tan\beta+i\sec\beta$. 5

(b) Prove that every square matrix satisfies its own characteristic equation. 7

MATHGE4-III

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

GROUP-A

1. Answer any ***four*** questions from the following: $3\times 4=12$

(a) Show that $x=1$ is a singular point of the ordinary differential equation.

$$(x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$$

(b) Evaluate: $\frac{1}{D^2+9}\sin 3x$

(c) Find all three solutions of $\frac{d^3y}{dx^3}-5\frac{d^2y}{dx^2}+8\frac{dy}{dx}-4=0$ and show that they are Linearly independent.

(d) Show that the derivative of a vector of constant length is perpendicular to the vector.

(e) A particle moves along a curve $x=e^{-t},\ y=2\cos 4t,\ z=2\sin 4t$, where t is time. Determine its velocity and acceleration at $t=\pi$.

(f) Evaluate ∇e^{r^2} where $r^2=x^2+y^2+z^2$.

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. Solve by Method of undetermined coefficients $\frac{d^2y}{dx^2}-3\frac{dy}{dx}=x+e^x\sin x$. 6

3. Solve: $\frac{d^4y}{dx^4}-2\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=x^3$ 6

4. Solve $\frac{d^2y}{dx^2}-y=x$ in powers of x. 6

5. Solve: 6

$$\frac{dy}{dx}+2y-3z=x$$

$$\frac{dz}{dx}+2z-3y=e^{2x}$$

6. If $\vec{F}=3xy\hat{i}-5z\hat{j}+10x\hat{k}$, evaluate $\int_C \vec{F}\cdot d\vec{r}$ along the curve C given by $x=t^2+1$, $y=2t^2,\ z=t^3$ from $t=1$ to $t=2$. 6

7. Find the directional derivative of the function $f=\frac{1}{(x^2+y^2+z^2)^{1/2}}$ at the point (3, 1, 2) in the direction of the vector $yz\hat{i}+zx\hat{j}+xy\hat{k}$. 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) Solve: $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}$ 6

(b) Solve by method of variation of parameters $\frac{d^2y}{dx^2}+a^2y=\tan ax$. 6

9. (a) Solve $x\frac{d^2y}{dx^2}-(x-2)\frac{dy}{dx}+2y=x^3e^x$ after determination of a solution of its reduced equation. 6

(b) Solve: $\frac{d^2x}{dt^2}+4x+y=te^{3t}$ 6

$\frac{d^2y}{dt^2}+y-2x=\cos^2 t$

10.(a) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ along the curve $x^2+y^2=1,\ z=1$ in the positive direction from (0, 1, 1) to (1, 0, 1) if $\vec{F}=(2x+yz)\hat{i}+xz\hat{j}+(xy+2z)\hat{k}$. 6

(b) If $u=x+y+z,\ v=x^2+y^2+z^2,\ w=xy+yz+zx$, prove that 6

$$(\text{grad}\,u)\cdot[(\text{grad}\,v)\times(\text{grad}\,w)]=0$$

11.(a) Prove that $\frac{d}{dt}\left(\vec{F}(t)\times\vec{G}(t)\right)=\vec{F}(t)\times\frac{d\vec{G}(t)}{dt}+\frac{d\vec{F}(t)}{dt}\times\vec{G}(t)$. 4

(b) Prove that $\text{grad}\,\phi$ is an irrotational vector field. 4

(c) State Green's theorem and show that it is a particular case of Stoke's theorem. 4

MATHGE4-V

NUMERICAL METHODS

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) If $\frac{5}{6}$ represent approximately by 0.8333. Find relative error and percentage error.

(b) Evaluate: $\left(\frac{\Delta^2}{E}\right)x$

(c) What is the geometrical representation of the Regula-Falsi method?

(d) State three differences between direct and iterative methods.

(e) Show that $\nabla y_{n+1}=h\left[1+\frac{1}{2}\nabla+\frac{5}{12}\nabla^2+\cdots\cdots\right]Dy_n$, where D is the differential operator.

(f) Prove that $(1+\Delta)(1-\nabla)=1$.

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. Explain the Secant method for numerical solution of the equation $f(x)=0$.

3. Use iterative formula to evaluate $\sqrt[7]{125}$, correct upto four significant figures.

4. Deduce Newton's backward interpolation formula with error terms.

5. Find $f(1.02)$ from the following table:

x	1.00	1.10	1.20	1.30
$f(x)$	0.8415	0.8912	0.9320	0.9636

6. The equation $x^2 + px + q = 0$ has two real roots α, β. Show that the iteration method $x_{k+1} = -\dfrac{px_k + q}{x_k}$ is convergent near $x = \alpha$, if $|\alpha| > |\beta|$.

7. Evaluate $\int_0^{\pi/2} \sqrt{\sin x}\, dx$, taking $n = 6$, correct upto four significant figures by Simpson's $\frac{1}{3}^{\text{rd}}$ rule.

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) Describe Newton's-Raphson method and find the geometrical interpretation of Newton's-Raphson method. 6

(b) Find the positive root of the equation $x^3 + x - 1 = 0$ by fixed point iteration method correct to three decimal places. 6

9. (a) Explain the 2nd order Runge-Kutta method for the numerical solution of a 1st order differential equation $\dfrac{dy}{dx} = f(x, y)$ subject to the initial condition $y = y_0$, when $x = x_0$. 6

(b) Solve by modified Euler's method, the following differential equation for $x = 1$ by taking $h = 0.2$; 6

$$\frac{dy}{dx} = xy \quad , \quad y = 1 \text{ when } x = 0$$

10.(a) Find a polynomial of least degree for the data $f(-1) = 1$, $f(0) = 1$, $f(1) = 1$ and $f(2) = -5$. 6

(b) Use Gauss-elimination method to solve the following system: 6

$$-10x_1 + 6x_2 + 3x_3 + 100 = 0$$
$$6x_1 - 5x_2 + 5x_3 + 100 = 0$$
$$3x_1 + 6x_2 - 10x_3 + 100 = 0$$

Correct upto three significant figures.

11.(a) Compute $\log_{10} 3.5$ from the data set: 6

x	2	3	5	7
$\log_{10} x$	0.301	0.477	0.699	0.845

(b) Compute the values of the unknowns in the system of equation by Gauss-Jordan's matrix inversion method: 6

$$6.1x_1 + 2.2x_2 + 1.2x_3 = 16.55$$
$$2.2x_1 + 5.5x_2 - 1.5x_3 = 10.55$$
$$1.2x_1 - 1.5x_2 + 7.2x_3 = 16.80$$

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GE2-P2-MATHEMATICS

(OLD SYLLABUS 2018)

Time Allotted: 2 Hours — Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains MATHGE4-I, MATHGE4-II, MATHGE4-III, MATHGE4-IV & MATHGE4-V. The candidates are required to answer any *one* from the *five* courses. Candidates should mention it clearly on the Answer Book.

MATHGE4-I

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GROUP-A

1. Answer any ***four*** questions: $3\times4 = 12$

(a) If $y=\cos^4 x$, then find y_n. 3

(b) Evaluate the following limit 3

$$\lim_{x\to 0}\frac{e^x+\sin x-1}{\log(1+x)}$$

(c) Find the centre and radius of the sphere $x^2+y^2+z^2+2x-4y-6z+5=0$. 3

(d) Find the point of inflexion of the curve $x=(y-1)(y-2)(y-3)$. 3

(e) Find the length of one arc of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$. 3

(f) Solve: $\dfrac{dy}{dx}=1+e^{x+y}$ 3

GROUP-B

Answer any *four* questions from the following $6\times4 = 24$

2. Find the envelope of the family of ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, where the parameters a and b are connected by $\sqrt{a}+\sqrt{b}=\sqrt{c}$ where c is a constant. 6

3. Find the asymptotes of the curve $y^4-2y^3x+2yx^3-x^4+x^2-y^2+x-y-1=0$. 6

4. Trace the curve $y^2=x^2(1-x^2)$. 6

5. If $I_n=\int\dfrac{t^n}{1+t^2}dt$, show that $I_{n+2}=\dfrac{t^{n+1}}{n+1}-I_n$. Hence find I_5. 6

6. Reduce the following equation to its canonical form and determine the nature of the conic represented by it: 6

$$4x^2-4xy+y^2+2x-26y+9=0$$

7. (a) Solve: $y^2\,dx + (x - \frac{1}{y})\,dy = 0$ 4+2

(b) Find the integrating factor of $(x^2 + xy^4)\,dx + 2y^3\,dy = 0$.

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) Evaluate: $\lim\limits_{x\to 0} (\cot x)^{1/\log x}$ 4

(b) Find $\int x^4 e^{ax}\,dx$. 4

(c) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. 4

9. (a) Solve the differential equation: $(xy^2 - e^{1/x^3})\,dx - x^2 y\,dy = 0$. 4

(b) Obtain the differential equation of all parabolas each of which has a latus rectum $4a$, and whose axes are parallel to the x-axis. 4

(c) Solve: $(y^2 e^{xy^2} + 4x^3)\,dx + (2xye^{xy^2} - 3y^2)\,dy = 0$. 4

10.(a) A plane passing through a fixed point (a, b, c) cuts the axes at A, B, C. Show that the locus of the centre of the sphere $OABC$ is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$. 6

(b) Find the equation of the right circular cylinder which passes through the point $(3, -1, 1)$ and has the line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{1}$ as axis. 6

11.(a) Find the condition that the line $\frac{l}{r} = a\cos\theta + b\sin\theta$ may touch the conic $\frac{l}{r} = 1 + e\cos(\theta - \beta)$. 6

(b) Show that the whole area of the curve $a^2 y^2 = x^3(2a - x)$ is πa^2. 6

MATHGE4-II

ALGEBRA

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) Prove that $1 + 2 + \cdots\cdots + n = \frac{n(n+1)}{2}$, $\forall n \in \mathbb{N}$, by the principle of induction. 3

(b) Show that the product of all values of $(1 + i\sqrt{3})^{3/4}$ is 8. 3

(c) Prove that an inverse of the equivalence relation is an equivalence relation. 3

(d) Find the number and nature of real roots of the equation: 3

$$x^4 + 4x^3 - x^2 - 2x - 5 = 0.$$

(e) Find all eigen values of the following matrix: $\begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$. 3

(f) Find the rank of the matrix $\begin{pmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 1 & 4 & 4 \end{pmatrix}$. 3

GROUP-B

Answer any *four* questions $6\times4=24$

2. Prove that $a^6+b^6+c^6+d^6 \geq abcd\,(a^2+b^2+c^2+d^2)$ where a, b, c, d are four positive real numbers. 6

3. Using the principle of induction, prove that $10^{n+1}+10^n+1$ is divisible by 3 for all $n\in\mathbb{N}$. 6

4. Solve the system of equations using row reduced form: 6

$$x+2y+z=1$$
$$3x+y+2z=3$$
$$x+7y+2z=1$$

5. Verify Cayley-Hamilton theorem for the following matrix: 6

$$A=\begin{pmatrix}1 & 0 & 2\\ 0 & -1 & 1\\ 0 & 1 & 0\end{pmatrix}$$

Hence find A^{-1} and A^9.

6. Solve: $2x^4+8x^3+3x^2+4x+1=0$, whose the sum of two roots is zero. 6

7. Solve by Ferrari's method: $x^4+4x^3-6x^2+20x+8=0$ 6

GROUP-C

Answer any *two* questions $12\times2=24$

8. (a) If α,β,γ be the roots of the equation $x^3+px^2+qx+r=0$, find the equation whose roots are $\beta^2+\gamma^2-\alpha^2,\ \gamma^2+\alpha^2-\beta^2,\ \alpha^2+\beta^2-\gamma^2$. 3

(b) Find the least positive residue in $2^{41}(\text{mod }23)$. 3

(c) Show that $a^{12}-b^{12}$ is divisible by 91 if a and b both are prime to 91. 6

9. (a) Reduce the matrix A to its row reduced echelon form and hence find its rank, 4

where $A=\begin{pmatrix}0 & 1 & -3 & 1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{pmatrix}$.

(b) If p is a prime and a, b are positive integers, then show that 4

$$(a+b)^p\equiv(a+b)\ (\text{mod } p).$$

(c) Find two integers u and v satisfying $54u+24v=30$. 4

10. (a) Solve $x^3-6x^2-6x-7=0$ by Cardon's method. 6

(b) If $\cos\alpha+\cos\beta+\cos\gamma=0$ and $\sin\alpha+\sin\beta+\sin\gamma=0$, prove that 6

$$\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$$

and $$\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$$

11. (a) Find $\cos2\theta\cosh2\phi$, if $\sin(\theta+i\phi)=\tan\beta+i\sec\beta$. 5

(b) Prove that every square matrix satisfies its own characteristic equation. 7

MATHGE4-III

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4 = 12$

(a) Show that $x=1$ is a singular point of the ordinary differential equation.

$$(x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$$

(b) Evaluate: $\frac{1}{D^2+9}\sin 3x$

(c) Find all three solutions of $\frac{d^3y}{dx^3}-5\frac{d^2y}{dx^2}+8\frac{dy}{dx}-4=0$ and show that they are Linearly independent.

(d) Show that the derivative of a vector of constant length is perpendicular to the vector.

(e) A particle moves along a curve $x=e^{-t},\ y=2\cos 4t,\ z=2\sin 4t$, where t is time. Determine its velocity and acceleration at $t=\pi$.

(f) Evaluate ∇e^{r^2} where $r^2=x^2+y^2+z^2$.

GROUP-B

Answer any *four* questions from the following $6\times4 = 24$

2. Solve by Method of undetermined coefficients $\frac{d^2y}{dx^2}-3\frac{dy}{dx}=x+e^x\sin x$. 6

3. Solve: $\frac{d^4y}{dx^4}-2\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=x^3$ 6

4. Solve $\frac{d^2y}{dx^2}-y=x$ in powers of x. 6

5. Solve: $\frac{dy}{dx}+2y-3z=x$ 6

$$\frac{dz}{dx}+2z-3y=e^{2x}$$

6. If $\vec{F}=3xy\hat{i}-5z\hat{j}+10x\hat{k}$, evaluate $\int_C \vec{F}\cdot d\vec{r}$ along the curve C given by $x=t^2+1$, $y=2t^2,\ z=t^3$ from $t=1$ to $t=2$. 6

7. Find the directional derivative of the function $f=\frac{1}{(x^2+y^2+z^2)^{1/2}}$ at the point (3, 1, 2) in the direction of the vector $yz\hat{i}+zx\hat{j}+xy\hat{k}$. 6

GROUP-C

Answer any *two* questions from the following $12\times2 = 24$

8. (a) Solve: $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}$ 6

(b) Solve by method of variation of parameters $\frac{d^2y}{dx^2}+a^2y=\tan ax$. 6

9. (a) Solve $x\frac{d^2y}{dx^2}-(x-2)\frac{dy}{dx}+2y=x^3e^x$ after determination of a solution of its reduced equation. 6

(b) Solve: $\frac{d^2x}{dt^2}+4x+y=te^{3t}$ 6

$$\frac{d^2y}{dt^2}+y-2x=\cos^2 t$$

10.(a) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ along the curve $x^2+y^2=1,\ z=1$ in the positive direction from (0, 1, 1) to (1, 0, 1) if $\vec{F}=(2x+yz)\hat{i}+xz\hat{j}+(xy+2z)\hat{k}$. 6

(b) If $u=x+y+z$, $v=x^2+y^2+z^2$, $w=xy+yz+zx$, prove that 6

$$(\text{grad}\, u)\cdot[(\text{grad}\, v)\times(\text{grad}\, w)]=0$$

11.(a) Prove that $\frac{d}{dt}\left(\vec{F}(t)\times\vec{G}(t)\right)=\vec{F}(t)\times\frac{d\vec{G}(t)}{dt}+\frac{d\vec{F}(t)}{dt}\times\vec{G}(t)$. 4

(b) Prove that $\text{grad}\,\phi$ is an irrotational vector field. 4

(c) State Green's theorem and show that it is a particular case of Stoke's theorem. 4

MATHGE4-IV

GROUP THEORY

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Prove that a group $(G,\circ)$ contains only one identity element. 3

2. Prove that in a group $G(ab)^{-1}=b^{-1}a^{-1}$ for all $a, b\in G$. 3

3. Show that the set of matrices 3

$$\left\{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}, \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\right\}$$

forms a commutative group under matrix multiplication.

4. Let a be an element of a group G. If $O(a)=n$ and $a^m=e$, then show that n is a divisor of m. 3

5. Prove that every cyclic group is abelian. 3

6. Let $(G,\circ)=(\mathbb{Z}, +)$ and a mapping $\phi: G\to G$ be defined by $\phi(x)=x+1$, $x\in G=\mathbb{Z}$. Examine if ϕ is a homomorphism. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. Let H and K be subgroups of a group G. Then show that HK is a subgroup of G if and only if $HK = KH$. 6

8. Prove that every group of prime order is cyclic. 6

9. Prove that a finite cyclic group of order n has one and only one subgroup of order d for every divisor d of n. 6

10. Let H be a subgroup of a group G. Then show that H is normal in G if and only if $h \in H$ and $x \in G \Rightarrow xhx^{-1} \in H$. 6

11. Let $f : G \to G'$ be a group homomorphism. Let $a \in G$ be such that $O(a) = n$ and $O(f(a)) = m$. Prove that $O(f(a)) \mid O(a)$ and f is one-one if and only if $m = n$. 6

12. Let H be a subgroup of a group. Prove that the relation ρ defined on G by "$a \rho b$ if and only if $a^{-1}b \in H$" for $a, b \in G$ is an equivalence relation on G. 6

GROUP-C

Answer any *two* questions from the following $12\times2 = 24$

13.(a) Let G be a finite cyclic group generated by a. Then show that $O(G) = n$ if and only if $O(a) = n$. 6

(b) Let G and G' be two groups and $\phi : G \to G'$ be a homomorphism, prove that $\phi(G)$ is a subgroup of G'. 6

14.(a) Let H be a subgroup of a group G. Prove that there is always a one-one onto mapping between any two right cosets of H in G. 6

(b) Let $f : G \to G'$ be a group homomorphism. Then show that $\ker f$ is a normal subgroup of G. 6

15.(a) If G is a finite group, then show that the order of any element of G divides the order of G and $a^{O(G)} = e$ for any $a \in G$. 7

(b) Let H be a subgroup of a group G and $[G : H] = 2$. Then show that H is normal in G. 5

16.(a) Let H be a subgroup of a commutative group G. Prove that G/H is commutative. Is converse true? Justify. 5+2

(b) Show that the map ϕ: $(\mathbb{Z}_5, +_5) \longrightarrow (\mathbb{Z}_5, +_5)$, defined by $\phi(\bar{x}) = 3\bar{x}$, $\bar{x} \in \mathbb{Z}_5$ is an isomorphism. 5

MATHGE4-V

NUMERICAL METHODS

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4 = 12$

(a) If $\frac{5}{6}$ represent approximately by 0.8333. Find relative error and percentage error.

(b) Evaluate: $\left(\frac{\Delta^2}{E}\right)x$

(c) What is the geometrical representation of the Regula-Falsi method?

(d) State three differences between direct and iterative methods.

(e) Show that $\nabla y_{n+1} = h\left[1 + \frac{1}{2}\nabla + \frac{5}{12}\nabla^2 + \cdots\cdots\right]Dy_n$, where D is the differential operator.

(f) Prove that $(1+\Delta)(1-\nabla) = 1$.

GROUP-B

Answer any *four* questions from the following $6\times4=24$

2. Explain the Secant method for numerical solution of the equation $f(x)=0$.

3. Use iterative formula to evaluate $\sqrt[7]{125}$, correct upto four significant figures.

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$f(x)$	0.8415	0.8912	0.9320	0.9636

6. The equation $x^2+px+q=0$ has two real roots α, β. Show that the iteration method $x_{k+1}=-\frac{px_k+q}{x_k}$ is convergent near $x=\alpha$, if $|\alpha|>|\beta|$.

7. Evaluate $\int_0^{\pi/2}\sqrt{\sin x}\,dx$, taking $n=6$, correct upto four significant figures by Simpson's $1/3^{\text{rd}}$ rule.

GROUP-C

Answer any *two* questions from the following $12\times2=24$

8. (a) Describe Newton's-Raphson method and find the geometrical interpretation of Newton's-Raphson method. 6

(b) Find the positive root of the equation $x^3+x-1=0$ by fixed point iteration method correct to three decimal places. 6

9. (a) Explain the 2nd order Runge-Kutta method for the numerical solution of a 1st order differential equation $\frac{dy}{dx}=f(x,\ y)$ subject to the initial condition $y=y_0$, when $x=x_0$. 6

(b) Solve by modified Euler's method, the following differential equation for $x=1$ by taking $h=0.2$; 6

$$\frac{dy}{dx}=xy\ ,\quad y=1\ \text{ when }\ x=0$$

10. (a) Find a polynomial of least degree for the data $f(-1)=1, f(0)=1, f(1)=1$ and $f(2)=-5$. 6

(b) Use Gauss-elimination method to solve the following system: 6

$$-10x_1+6x_2+3x_3+100=0$$
$$6x_1-5x_2+5x_3+100=0$$
$$3x_1+6x_2-10x_3+100=0$$

Correct upto three significant figures.

11. (a) Compute $\log_{10}3.5$ from the data set: 6

x	2	3	5	7
$\log_{10}x$	0.301	0.477	0.699	0.845

(b) Compute the values of the unknowns in the system of equation by Gauss-Jordan's matrix inversion method: 6

$$6.1x_1+2.2x_2+1.2x_3=16.55$$
$$2.2x_1+5.5x_2-1.5x_3=10.55$$
$$1.2x_1-1.5x_2+7.2x_3=16.80$$

——×——